Northampton High School

Math Department

Summer Work for

AP Calculus

Show all work!!! Answers without justification are unacceptable

Functions

A **function** is a rule that assigns to each member in its **domain** a unique member in the **range.**

Example: $f(x) = x^2(x-3)(x+2)$

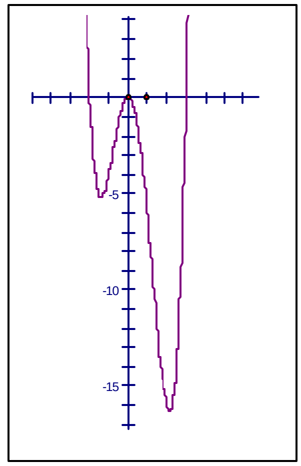
For domain value of -1, the range value would be -4, written $f(-1) = -4$

The **zeros (x-intercepts)** of $f(x)$ are $-2, 0, 3$ (set $y = 0$)

The **y-intercept** (set $x = 0$) of $f(x)$ is 0

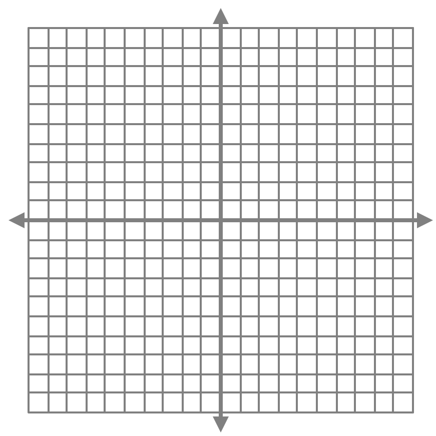
The **domain** of $f(x)$ is all real numbers

The **range** of $f(x)$ is all real numbers



Complete the following exercises:

1. Given: $f(x) = -|x+3| - 2$



a. Graph $f(x)$.

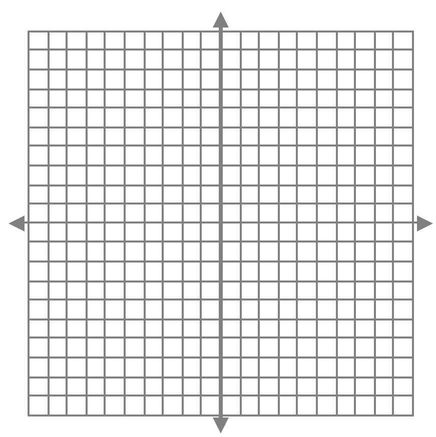
b. Domain: ___________

c. Range: ___________

d. $f(3) =$ ___________

e. If $f(x) = -3$, then $x =$ ___________

2. Given the piecewise function: $g(x) = \begin{cases} x/2 \text{, if } x \geq 4 \\ \sqrt{x} \text{, if } 0 < x < 4 \\ x^2 \text{, if } x < 0 \end{cases}$



a. Graph $g(x)$.

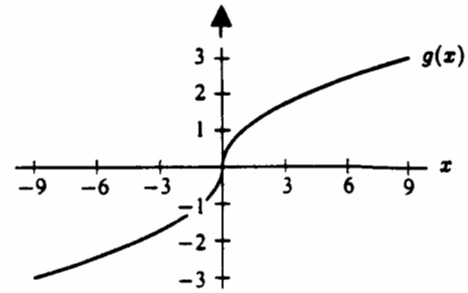
b. $g(-3) =$ _________

c. $g(1) =$ _________

d. $g(0) =$ _________

e. Is $g(x)$ a continuous function? Explain.

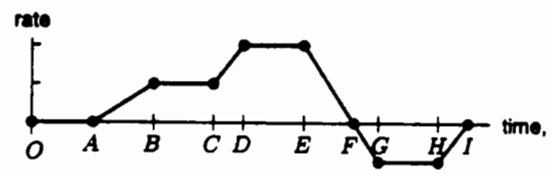
3. Given the graph of $g(x)$ on the right



a. Estimate $\frac{g(6)-g(0)}{6-0}$

b. The ratio in part (a) is the slope of a line segment joining two points on the graph. Sketch this line segment.

4. The rate at which water is entering a tank $(t>0)$ is represented by the given graph. A negative rate means that water is leaving the tank. State the interval(s) on which each of the following holds true:



a. The volume of water is constant ________________

b. The volume of water is decreasing ________________

c. The volume of water is increasing ________________

d. The volume of water is increasing fastest ________________

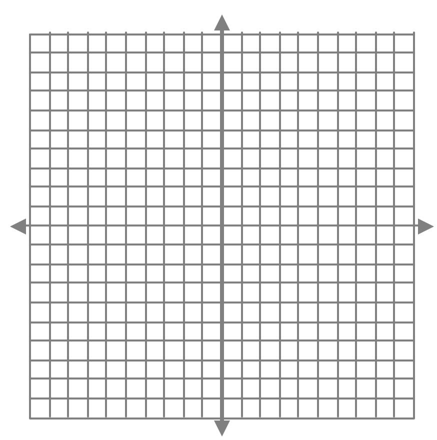
5. $Q(x)=\frac{3x}{x+1}$

a. Where is this function discontinuous? _______

b. State the equation of the vertical asymptote _______

c. State the equation of the horizontal asymptote _______

d. Sketch



e. Write the equation of the inverse of Q(x) _____________

TRIG TOPICS

1. The natural real number domain of the sine function is ________________; its range is ____________________.

2. The period of the cosine function is __________________; the period of the sine function is ______________________; the period of the tangent function is ______________.

3. Of the six trigonometric functions, _________________are odd functions and _____________________ are even functions.

4. If $(-4, 3)$ lies on the terminal side of an angle θ whose vertex is at the origin and initial side is along the positive x-axis, then $\cos q =$ __________(fraction) .

5. Convert the following degree measures to radians (leave π in your answer).

(a) 240^{o} = ______ (b) -60^{o} = ______ (c) -135^{o} = ______ (d) 540^{o} = ______

(e) 600^{o} = ______ (f) 720^{o} = ______ (g) 18^{o} = ______ (h) 22.5^{o} = ______

6. Convert the following radian measures to degrees.

(a) $\frac{7p}{6}$ = ______ (b) $\frac{-p}{3}$ = ______ (c) 8π = ______ (d) $\frac{5p}{4}$ = ______

(b) $\frac{3p}{2}$ = ______ (f) $\frac{-11p}{12}$ = ______ (g) $\frac{p}{18}$ = ______ (h) $\frac{7p}{4}$ = ______

7. Evaluate without use of a calculator.

(a) $\tan\left(\frac{p}{6}\right)$ = ______ (b) $\cos\left(\frac{-p}{3}\right)$ = ______ (c) $\sin(p)$ = ______ (d) $\csc\left(\frac{p}{2}\right)$ = ______

8. Verify that the following are identities:

(a) $(1+\sin z)(1-\sin z)=\frac{1}{\sec^{2} z}$

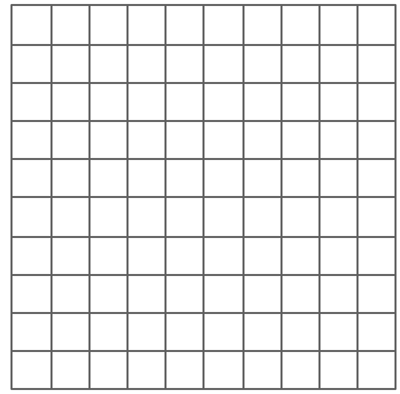
(b) $\frac{\sin u}{\csc u}+\frac{\cos u}{\sec u}=1$

(c) $\sec t-\sin t\tan t=\cos t$

9. Find identities analogous to the addition identities for each expression.

(a) $\sin(x-y)$ (b) $\cos(x-y)$ (c) $\tan(x-y)$

10. Draw the graphs of each of the following functions on $[-p\ ,\ 2p]$ using the same axis.



(a) $f(x) = \sin x$ (b) $f(x) = 2\cos x$ (c) $f(x) = \tan \frac{x}{2}$

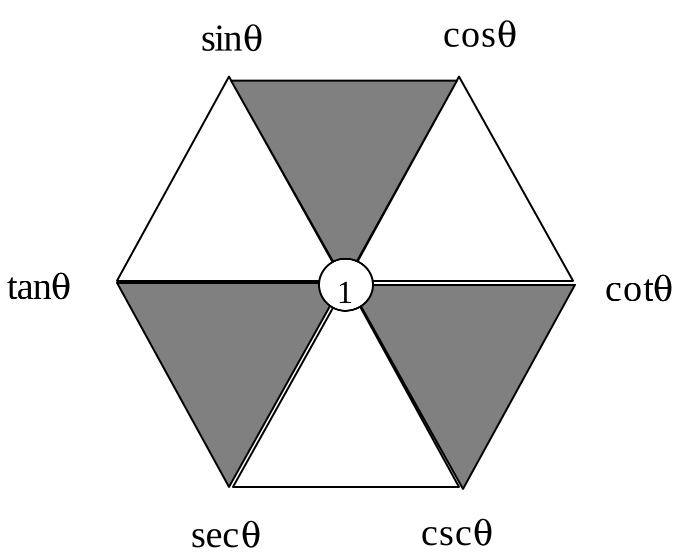
11. Find the exact values without use of a calculator.

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ ______ (b) $\cos^{-1}\left(\frac{1}{2}\right) =$ ______ (c) $\arcsin\left(-\frac{\sqrt{2}}{2}\right) =$ ______

(d) $\arctan\left(-\sqrt{3}\right) =$ ______ (e) $\sec^{-1}(-2) =$ ______ (f) $\sin\left(\cos^{-1}(0.6)\right) =$ ______

EIGHT FUNDAMENTAL IDENTITIES

Reciprocal Identities	**Ratio Identities**	**Pythagorean Identities**
1. $\csc q = \frac{1}{\sin q}, \sin q \neq 0$	**2.** $\tan q = \frac{\sin q}{\cos q}, \cos q \neq 0$	**3.** $\sin^2 q + \cos^2 q = 1$
4. $\sec q = \frac{1}{\cos q}, \cos q \neq 0$	**5.** $\cot q = \frac{\cos q}{\sin q}, \sin q \neq 0$	**6.** $1 + \tan^2 q = \sec^2 q$
7. $\cot q = \frac{1}{\tan q}, \tan q \neq 0$		**8.** $1 + \cot^2 q = \csc^2 q$



DERIVATIVES

The derivative is the limit of the slopes of a lot of secant lines. The limit of the slopes of the secant lines is the slope of the tangent line to the curve at a single point. Therefore, by transitivity, the "derivative of a function at a point is the slope of the tangent line at that single point. The notation $f'(x)$ is used to denote derivative. The derivative is a function just like $f(x)$, and is composed of ordered pairs $(x,\ f'(x))$. The formal definition of the derivative is a "limit" and is written:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The Power Theorem is a method used for taking the derivative of a variable power.

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

EXAMPLES: If $f(x) = x^4$, then $f'(x) = 4x^3$

If $f(x) = x^{3/2}$, then $f'(x) = 3/2\, x^{1/2}$

If $f(x) = x^{0.9}$, then $f'(x) = 0.9x^{-0.1}$

PROBLEMS:

A. Find the derivatives of the following functions using the definition of derivative.

1) $f(x) = 2x$ 2) $f(x) = x^2$ 3) $f(x) = \frac{1}{x}$

B. Find the derivatives of the following functions using the power theorem.

1) $f(x) = x^3$ 2) $f(x) = \sqrt{x}$ 3) $f(x) = 6x^3$

4) $f(x) = x^{4/3}$ 5) $f(x) = x^{-4}$ 6) $f(x) = \frac{1}{x^3}$

Logarithms

Definition of Logarithm: $y = \log_b x \Leftrightarrow b^y = x$

Log of a product: $\log_b(xy) = \log_b x + \log_b y$

Log of a quotient: $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

Log of a power: $\log_b x^n = n \log_b x$

Change - of - Base: $\log_a x = \frac{\log_b x}{\log_b a}$

Problems: Find x. Show work.

4. $\log_2 x = 3$

5. $\log_{\frac{1}{2}} x = 4$

6. $\log_3 81 = x$

7. $\log_3(-9) = x$

8. $\log_x 16 = -4$

1. $\log_x\left(\frac{1}{25}\right) = \frac{1}{2}$

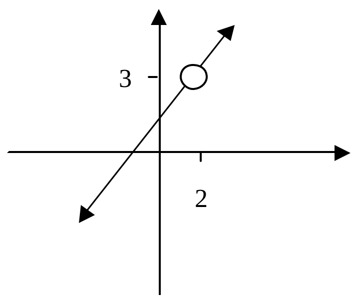
2. $2^x = 3$

3. $2.43 \bullet 10^x = 1.84$

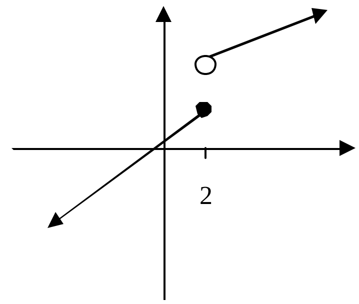
9. $\ln(x+5) = \ln(x-1) - \ln(x+1)$

Limits

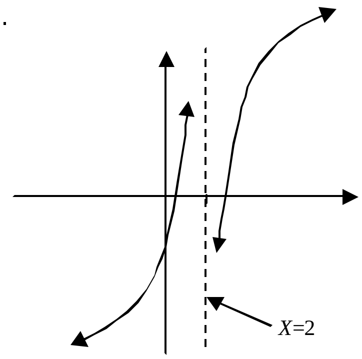
The limit of a function is the y-value that you are getting close to as x gets close to some number in the domain. In the "limit process" you never get to the limit, except for the limit of a constant function. We write $\lim_{x \to a} f(x)$, which is read "the limit of $f(x)$ as x approaches a domain value of a." The limit must be the same as x approaches "a" from both the left and the right. To find the limit, substitute in values very close to "a" on both left and right and see if the y-value is approaching a single value. The limit does exist at a hole in a graph, but does not exist at a vertical asymptote or a jump in the graph.



Limit exists at 2



No limit at 2

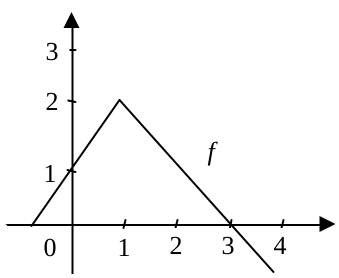


No limit at 2

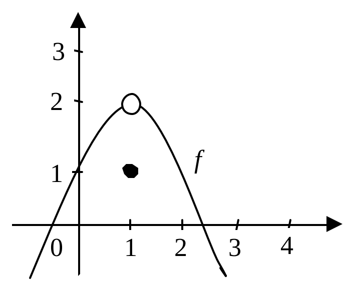
Problems:

The graphs of some functions are pictured below. Do you think that $\lim_{x \to 1} f(x)$ exists? If you think the limit does exist, state its value.

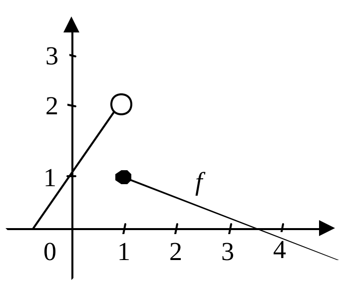
1.



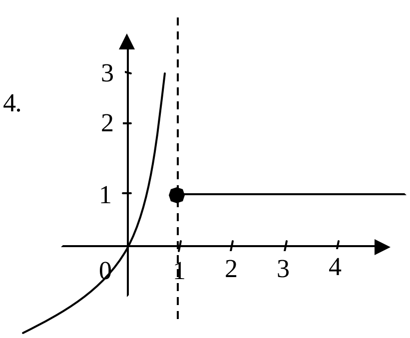
2.



3.



4.



State the value of each of the following:

5. $\lim_{x \to 3} x =$ ________

6. $\lim_{x \to -2} x =$ ________

7. $\lim_{x \to 1} 2x =$ ________

8. $\lim_{x \to -1} (x+1) =$ ________

9. $\lim_{x \to -2} x^2 =$ ________

10. $\lim_{x \to -2} |x| =$ ________